# Bouncing Ball (CAV 2010)

AVACS H4
Phase 2

July 28, 2011

## 1 Description of the Benchmark

This case study is related to our CAV paper [2] and models a bouncing ball assembled from different materials. Assume this ball consisting of three parts: 50 per cent of its surface consists of hard material, 25 per cent consists of a material of medium hardness and the rest consists of very soft material. We drop the ball from a height $h = 2$. Further, we assume the gravity constant to be $g = 1$. If the ball hits the floor and falls on the hard side, it will jump up again with one half of the speed it had before, if it falls down on the medium side it will jump up with a quarter of the speed. However, if it hits the ground with its soft side, it won't jump up again. We assume that the side with which the ball hits the ground is determined randomly, according to the amount surface covered by each of the materials.

We study the probability that the ball falls on the soft side before time bound $T$ and consider several time bounds. We conducted three main analysis settings. Two of them used partitioning with interval length of 0.15 on the position and speed variables, with the difference that *PHAVer*-specific convex hull over-approximation [1] was additionally used in the second one. For the third setting, we used an interval length of 0.05 and the convex hull over-approximation.

## 2 Results

We implemented all configurations in *PHAVer* format [1] and applied *ProHVer* [1] to them. The results are given in Table 1. Entries for which the analysis did not terminate within one hour are marked by "-".

We ascertain here that, without the convex hull over-approximation, with an interval of length 0.15, we obtain non-trivial upper bounds. However, the analysis time as well as the number of states grows very fast with increasing time bound. The reason for this to happen is, that each time the ball hits the ground, there are three possibilities with which side it will hit the ground. Thus, the number of possibilities for the amount of energy the ball still has after the ground is hit is exponential in n. Also notice that there

[1] `http://depend.cs.uni-sb.de/tools/prohver`

| *Time bound* | Interval length 0.15 | | |
|---|---|---|---|
| | Probability | Build(s) | Abstract states |
| 1 | 0 | 1 | 38 |
| 2 | 0.25 | 3 | 408 |
| 3 | 0.5 | 13 | 3000 |
| 3.5 | 0.5 | 70 | 11000 |
| 3.6 | 0.5 | 140 | 14000 |
| 3.7 | 0.5 | 214 | 18000 |

| *Time bound* | Interval length 0.15, hull | | |
|---|---|---|---|
| | Probability | Build(s) | Abstract states |
| 1 | 0 | 1 | 17 |
| 2 | 0.25 | 2 | 59 |
| 3 | 0.5 | 4 | 124 |
| 3.5 | 1 | 5 | 145 |
| 3.6 | 1 | 6 | 150 |
| 3.7 | 1 | 6 | 154 |

| *Time bound* | Interval length 0.05, hull | | |
|---|---|---|---|
| | Probability | Build(s) | Abstract states |
| 1 | 0 | 2 | 56 |
| 2 | 0.25 | 9 | 185 |
| 3 | 0.312 | 22 | 347 |
| 3.5 | 0.5 | 32 | 425 |
| 3.6 | 0.5 | 37 | 436 |
| 3.7 | - | - | - |

Table 1: Performance statistics for bouncing ball

is some time bound up to which the ball has done an infinite number of jumps in all cases, as this case study features Zeno behaviour. This indicates that for time bounds near this value we have to use very small partitioning intervals to obtain realistic probability bounds. This leads to an even larger time and memory consumption in the analysis.

If we use the convex hull over-approximation and an interval length of 0.15, far less resources have to be used. But we only obtain non-trivial results for time bounds up to 3. Using an interval width of 0.05, we obtain a tighter probability bound, while using still less resources than the first configuration. However, for time bound $T = 3.7$ the third configuration does not terminate within one hour.

## References


[1] Goran Frehse. PHAVer: Algorithmic Verification of Hybrid Systems Past HyTech. In Manfred Morari and Lothar Thiele, editors, *Hybrid Systems: Computation and Control*, volume 3414 of *LNCS*, pages 258–273. Springer, 2005.

[2] Lijun Zhang, Zhikun She, Stefan Ratschan, Holger Hermanns, and Ernst Moritz Hahn. Safety Verification for Probabilistic Hybrid Systems. In *CAV*, volume 6174 of *LNCS*, pages 196–211. Springer, 2010.