Autonomous Lawn-Mower (CAV 2010)

AVACS H4
Phase 2

July 28, 2011

1 Description of the Model

We consider an autonomous lawn-mower that uses a probability bias to avoid patterns on lawns. This mower is started on a rectangular grassed area. When it reaches the border of the area, it changes its direction. To prevent the mower from following a simple cycle pattern, this direction is randomly biased, such as to ensure that finally the entire area has been cut. A sketch of the automaton is given in Figure 1. There, l is

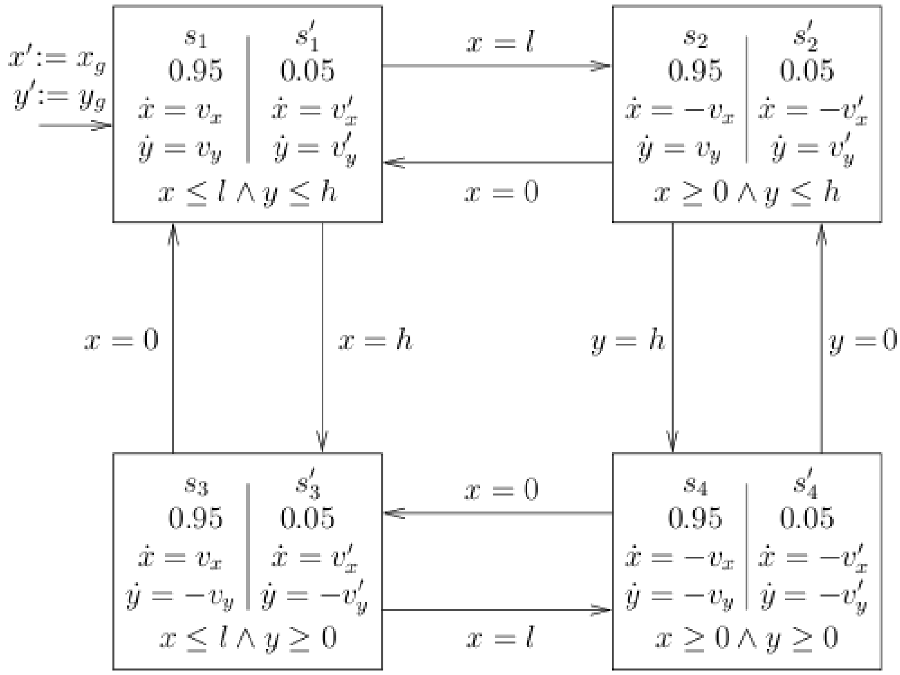

Figure 1: Model of the Lawn-Mower

the length and h the width of the area. The position of the mower on the area is given by (x, y). With (v_x, v_y) we denote the speed in (x, y) direction, which the mower takes with probability 0.95 when reaching a border, whereas (v'_x, v'_y) denotes a variation of the

Time bound	Probability	Build (s)	Abstract states
10	0	0.06	4
70	1.11984E-05	1.22	632
100	1.11984E-05	8.47	3022
110	0.000281861	65.55	9076
120	0.000281861	98.83	12660
130	0.000281861	303.40	25962
140	0.000281861	743.43	38830

Table 1: Performance Statistics for Lawn-Mower

speed that is taken with probability 0.05. Further, (x_g, y_g) describes the mower's initial position. The previously described case study is related to our CAV paper [2]

At the region with $x \geq 90 \wedge x \leq 100 \wedge y \geq 170 \wedge y \leq 200$ the owner of the lawn has left a tarpaulin. We are interested in the probability that the mower hits the tarpaulin within a time bound of $t = 120$ and thereby inevitably ripping the tarpaulin up.

For the analysis, we set $v_x = 10$, $v_y = 10$, $v'_x = 11$, $v'_y = 9$, $l = 100$, $h = 200$, $x_g = 10$ and $y_g = 20$.

2 Results

The creation of the labelled transition system for this automaton took 98 seconds whereas the computation time of the failure probability was negligible. The upper bound we obtained was 0.000281861. We did not use any interval specifications. All results were obtained by applying *ProHVer* [1] to the model.

The results of our analysis considering various time bounds are given in Table 1. Due to the fact that each time the mower reaches a border it may head into two different directions, the analysis time as well as the number of states grows quickly for larger time bounds. The choice of two directions lead to a combinatorial explosion, evident by the given statistics.

We also experimented with convex-hull overapproximations [1] without interval refinement. However, the probability bounds obtained were always 1. Using interval partitioning also did not improve on this, as it only made the analyses take more time. We feel that, for this case study, there is not much hope of obtaining better results in resource usage. The complexity does not really live in the hybrid behaviour, but results from the excessive number of ways the mower can pass around the are. Because of this, we don't think it is possible to find an abstraction to handle this case study for larger time bounds.

[1] `http://depend.cs.uni-sb.de/tools/prohver`

References

[1] Goran Frehse. PHAVer: Algorithmic Verification of Hybrid Systems Past HyTech. In Manfred Morari and Lothar Thiele, editors, *Hybrid Systems: Computation and Control*, volume 3414 of *LNCS*, pages 258–273. Springer, 2005.

[2] Lijun Zhang, Zhikun She, Stefan Ratschan, Holger Hermanns, and Ernst Moritz Hahn. Safety Verification for Probabilistic Hybrid Systems. In *CAV*, volume 6174 of *LNCS*, pages 196–211. Springer, 2010.