

# Random Walk

AVACS S3

Phase 2

July 28, 2011

In this test case, we consider a discrete-space, continuous-time random walk model, in which a walker starts from the initial position 0 and changes position with rate  $\lambda$ . The field directly left to the current position is chosen with probability  $p$  and the field to the right with probability  $1 - p$ . We consider two properties:

1. an instantaneous reward property: The expected distance from the starting point at a given point in time **and**
2. a probabilistic reachability property: The probability that a walker moves at least 10 fields to the right within a certain time bound  $t$ , expressed by  $P_{=?}(F^{\leq t} m \geq 10)$  where  $m$  denotes the position of the walker.

We applied *INFAMY* [1] to this test case for  $p = 0.25$  and  $\lambda = 1$ . For the truncation, a precision of  $10^{-6}$  was used. Table 1 gives a comparison of three tested configurations, Uniform, Layerd-chain and FSP. The results of the first (exp.) and second property (prob.) are given in Table 2. All values were obtained on a Linux machine with an AMD Athlon XP 2600+ processor at 2 GHz equipped with 2 GB of RAM.

Here, the analysis time given is only for analyzing the first property after state-space exploration. The time with respect to the second property is in the same order as the first one. As apparent from the data, the size of the model is approximately proportional to the depth and the time bound  $t$ . The Layered-chain and FSP configurations achieve consistently smaller truncation depths than Uniform. Due to the simple structure of the model, however, FSP turns out to be slower than the Uniform and Layered-chain configurations. This is because, for the FSP configuration, the transient analysis for state exploration consumes a proportionally high amount of time, especially for large time bounds  $t$ .

## References

- [1] H. Hermanns, E. M. Hahn, B. Wachter, and L. Zhang. Time-Bounded Model Checking of Infinite-State Continuous-Time Markov Chains. *Fundamenta Informaticae*, 95:129–155, 2009.

$t$	Uniform			Layered			FSP		
	dep	time (s)	$n$	dep	time (s)	$n$	dep	time (s)	$n$
50	223	0.9/0.0	447	71	0.9/0.0	143	62	0.9/0.0	125
100	273	0.9/0.0	547	121	0.9/0.0	243	101	1.0/0.0	203
1K	1.3K	0.9/0.1	2.5K	885	0.9/0.1	1.8K	654	16.3/0.0	1.3K
5K	5.6K	0.9/1.8	11.2K	4.0K	1.2/1.2	8.1K	2.8K	1462/0.7	5.7K

Table 1: Comparison of Uniform, Layered-chain and FSP configurations

$t$	exp.	prob.
50	2.50E+01	0.99
100	5.00E+01	1.00
1K	5.00E+02	1.00
5K	2.50E+03	1.00

Table 2: Results for the first and second property