Stochastic Job Scheduling

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1 Description of the Case Study

We consider a case study from the field of stochastic scheduling in which a number of tasks with exponentially distributed processing times have to be scheduled on two indentical processors [2]. Jobs may be preempted at decision points, i.e. the scheduler may temporarily interrupt a task, resuming another one and eventually finishing the task at a later time. Simple scheduling strategies such as "longest expected time first" suffice for minimizing the expected makespan, i.e. the completion time of the last job. We are instead interested in the maximal probability that all tasks are completed before a pre-specified deadline is reached. Notably, modeling this well-studied problem in stochastic scheduling naturally leads to non-uniform CTMDP. The model is rather small, having only 24 states.

2 Results

Analysis results obtained with MRMC are summarized in Table 1 using an algorithm to compute the maxima over time-dependent schedulers using a recent algorithm [3] and when using another algorithm [1] for the time-abstract, history-dependent case which

Time Bound	Time-Dependent Algorithm		Time-Abstract Algorithm	
	Time	Probability	Time	Probability
0.1	0s	0.0039984	6s	0.0039984
0.2	0s	0.0374710	42s	0.0374710
0.3	0s	0.1139450	3m 21s	0.1139450
0.4	0s	0.2218362	10m 17s	0.2218362
1.0	0s	0.7849833	-	-
2.0	0s	0.9787065	-	-
3.0	0s	0.9973928	-	-

Table 1: Statistics for the stochastic job scheduling. We give both number for the time-abstract algorithm as well as for the time-dependent algorithm

has a rather bad worst-case complexity. Since the model is not uniform, the efficient algorithm for this model class [4] could not be used.

For this case study, manual analysis shows that stationary scheduler indeed suffice to obtain maximal probabilities. Because of this, both algorithms lead to the same results. Notice that the time needed in the time-abstract case is much worse than for the time-dependent case. The same is true for memory requirements, not shown here. Indeed, for entries marked by "-", we were unable to obtain results because more than 2GB of main memory would have been needed. Although the large memory requirements for the time-abstract case may seem strange for this small model, they are in accordance with complexity results [1]. However, in the future we are planning to improve our implemenation of this algorithm such that the worst case is avoided when this is possible.

References

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