# Tandem Queueing Network

AVACS S3
Phase 2

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## 1 Description of the Model

This model [2] consists of two sequentially interconnected queues building a tandem queue. Each of them has capacity $c$. The network is represented as a CTMC consisting of a **M/Cox$_\mathbf{2}$/1-queue** as the first and a **M/M/1-queue** as the latter queue. The Coxian distribution representing the service time distribution is a special case of phase-type distribution. In this case, we consider two independent phases each of them representing an exponential distributed random variable with paramter $\mu_1$ and $\mu_2$, respectively. According to our implementation, the first queue either switches to phase 2 with rate $\mu_{1a}$ or synchronizes with the other queue and pushes an element into it with rate $\mu_{1b}$. If the phases were switched, both queues synchronize with rate $\mu_2$, an element is pushed from the first to the second one and the phase will be switched back to 1. A sketch of the model is depicted in Figure 1.

We consider the probability that the first queue becomes full before time 0.23, that is $P_{=?}(F^{\leq 0.23} sc = c)$. Larger time bounds are left out, since then this probability is one.

## 2 Results

We applied *INFAMY* [1], using the configuration Layered, to our model and compared the results with the ones of the corresponding *PRISM* case study [3]. As can be seen in Table 1, the performance results are significantly better using *INFAMY* since *PRISM* needs to explore almost the whole state space. Using our tool, we just need to explore

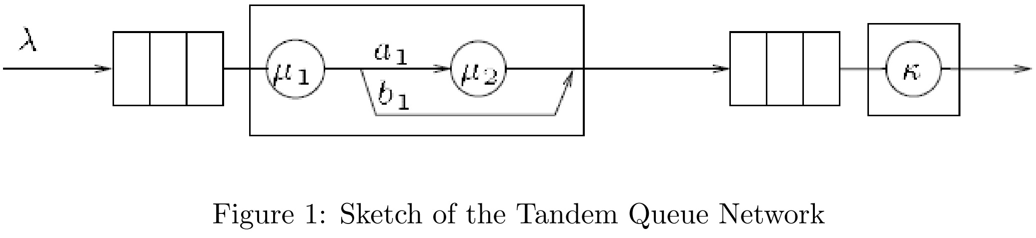

Figure 1: Sketch of the Tandem Queue Network

| $c$ | *PRISM* | | | Layered | | | Prob. |
|---|---|---|---|---|---|---|---|
| | depth | time | $n$ | depth | time | $n$ | |
| 511 | 1535 | 3.7/49.7 | 523776 | 632 | 6.8/7.5 | 235339 | 3.13E-02 |
| 1023 | 3071 | 13.7/380.3 | 2096128 | 1167 | 3.6/49.6 | 714306 | 4.24E-03 |
| 2047 | 6143 | 69.3/3068.3 | 8386560 | 2198 | 10.0/2978 | 2449798 | 9.87E-05 |
| 4095 | 12287 | 560.9/31386.5 | 33550336 | 4209 | 27.4/2889.8 | 8899113 | 7.06E-08 |

Table 1: Performance Statistics for Tandem Queues

a half of it in the worst case. However, for large time bounds ($t \geq 1$) the whole model will be explored and *PRISM* will perform better. The results were obtained on a Linux machine with an AMD AthlonTM XP 2600+ processor at 2 GHz equipped with 2 GB of RAM. The case study can also be found in [1]

## References

[1] Ernst Moritz Hahn, Holger Hermanns, Björn Wachter, and Lijun Zhang. INFAMY: An Infinite-State Markov Model Checker. In *CAV*, pages 641–647, 2009.

[2] H. Hermanns, J. Meyer-Kayser, and M. Siegle. Multi Terminal Binary Decision Diagrams to Represent and Analyse Continuous Time Markov Chains. In B. Plateau, W. Stewart, and M. Silva, editors, *NSMC*, pages 188–207, 1999.

[3] J.-P. Katoen, M. Kwiatkowska, G. Norman, and D. Parker. Faster and Symbolic CTMC Model Checking. In L. de Alfaro and S. Gilmore, editors, *Proc. 1st Joint International Workshop on Process Algebra and Probabilistic Methods, Performance Modeling and Verification (PAPM/PROBMIV'01)*, volume 2165 of *LNCS*, pages 23–38. Springer, 2001.