Using decision procedures for rich data structures for the verification of real-time systems

Viorica Sofronie-Stokkermans

Joint work with Johannes Faber, Carsten Ihlemann, Swen Jacobs and with Werner Damm, Matthias Horbach

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Problem statement

We consider parametric real time (infinite state) systems
– parametric data, parametric change, parametric topology of the system

Examples:

\[ n \text{ (number of trains)}; \quad l_{\text{alarm}} > 0; \quad 0 < v_{\text{min}} < v_{\text{max}}; \quad \ldots \]
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Main results

Our work in AVACS (R1)

- **Specification of systems with a complex topology**
  data structures (arrays, pointer structures)

- **Deductive verification**: Invariant checking, BMC, Constraints on parameters
  (using decision procedures for rich data structures, quantifier elimination)
  
  [Jacobs, VS: PDPAR'06, ENTCS'07], [Faber, Jacobs, VS: IFM'07],
  [Faber, Ihlemann, Jacobs, VS: IFM'10], [VS: IJCAR'10], [VS: CADE'13]

→ Efficient decision procedures for data structures
  
  local theory extensions [VS: CADE'05, FroCoS'07]
  - ordered structures [Ihlemann, VS: ISMVL’07]
  - theories of arrays & pointers [Ihlemann, Jacobs, VS: TACAS’08]
  - theories from mathematical analysis [VS: KI’08]
  - combinations of local theory extensions [Ihlemann, VS: IJCAR’10], [VS: PL’13]

→ Interpolation in local theory extensions ↔ CEGAR

  [VS: IJCAR’06, LMCS’08], [Rybalchenko, VS: VMCAI’07, JSC’10], [VS: PL’13]
State of the art/Main results

We consider parametric real time and hybrid (infinite state) systems – parametric data, parametric change, parametric topology

**Previous work** often only few aspects of parametricity studied together approximations/abstraction

Before [Jacobs, VS’06, ’07], [Faber, Jacobs, VS’07], [Faber, Ihlemann, Jacobs, VS’10]:
- only parametricity in the data domain: [Platzer, Quesel’09]
- parametric number of components:

Before [VS: CADE’13], [Damm, Horbach, VS: FroCoS’15]:
- modularity and small model property results for restricted classes of systems
  - [Kaiser, Kroening et al.’10], [Johnson, Mitra’12], [Abdulla et al’13]
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Our main goal: Reduce complexity by exploiting modularity at various levels:
specification / verification / structurally
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Our main goal: Reduce complexity by exploiting modularity at various levels:
specification / verification / structurally
Example 1: Verification of systems of trains

[Faber, Ihlemann, Jacobs, VS 2010]

Main goal: exploit modularity at various levels

1. Specification
   - Use the modular language COD, which allows us to separately specify
     - processes (as Communicating Sequential Processes, CSP),
     - data (using Object-Z, OZ), and
     - time (using the Duration Calculus, DC).
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1. Specification
   - Use the modular language COD, which allows us to separately specify
     - processes (as Communicating Sequential Processes, CSP),
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     - time (using the Duration Calculus, DC).

2. Verification
   - Verification tasks: invariant checking.
     \(\Rightarrow\) Problem: reasoning in complex data structures
     \(\Rightarrow\) Solution: hierarchical and modular reasoning
   - Use of COD allows us to decouple:
     \(\Rightarrow\) Verification tasks concerning data (OZ)
     \(\Rightarrow\) Verification tasks concerning durations (DC)

Allows us to impose/verify conditions on the single components which guarantee safety of the overall system.
Main goal: exploit modularity at various levels

3. Structurally

• Running example: Complex track topologies
Main goal: exploit modularity at various levels

3. Structurally

- **Running example**: Complex track topologies

  - One line track: Verification
  - Complex track topology:
    - decomposition into family of linear tracks
    - prove that safety of whole system follows from safety for the controller of a linear track.
Overview

• Modular Specifications: COD

• Modular Verification

• Modularity at structural level

• Implementation; experimental results

• Conclusions
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Modular Specifications: CSP-OZ-DC (COD)

COD [Hoenicke, Olderog’02] allows us to specify in a modular way:

- the control flow of a system using Communicating Sequential Processes (CSP)
- the state space and its change using Object-Z (OZ)
- (dense) real-time constraints over durations of events using the Duration Calculus (DC)
COD [Hoenicke, Olderog’02] allows us to specify in a modular way:

- the control flow of a system using Communicating Sequential Processes (CSP)
- the state space and its change using Object-Z (OZ)
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**Benefits:**

- **Compositionality:** it suffices to prove safety properties for the separate components to prove safety of the entire system
- **high-level tool support** given by Syspect (easy-to-use front-end to formal real-time specifications, with a graphical user interface).
Example: Controller for line track (RBC)
**CSP part**: specifies the processes and their interdependency.

The RBC system passes repeatedly through four phases, modeled by events:

- **updSpd** (speed update)
- **req** (request update)
- **alloc** (allocation update)
- **updPos** (position update)

Between these events, trains may leave or enter the track (at specific segments), modeled by the events **leave** and **enter**.
Example: Controller for line track (RBC)

**CSP part:** specifies the processes and their interdependency.

The RBC system passes repeatedly through four phases, modeled by events with corresponding COD schemata:

**CSP:**

method `enter` : `[s1? : Segment; t0? : Train; t1? : Train; t2? : Train]`

method `leave` : `[ls? : Segment; lt? : Train]`

local chan `alloc, req, updPos, updSpd`

main `≡` ((`updSpd` `→` State1)  State1 `≡` ((`req` `→` State2)  State2 `≡` ((`alloc` `→` State3)  State3 `≡` ((`updPos` `→` main)

  □ (leave `→` main)  □ (leave `→` State1)  □ (leave `→` State2)  □ (leave `→` State3)

  □ (enter `→` main)  □ (enter `→` State1)  □ (enter `→` State2)  □ (enter `→` State3)
Example: Controller for line track (RBC)

OZ part. Consists of data classes, axioms, the Init schema, update rules.
Example: Controller for line track (RBC)

**OZ part.** Consists of data classes, axioms, the \texttt{Init} schema, update rules.

- **1. Data classes** declare function symbols that can change their values during runs of the system

**Data structures:**

- **2-sorted pointers**

```
<table>
<thead>
<tr>
<th>SegmentData</th>
<th>TrainData</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{train} : \text{Segment} \rightarrow \text{Train} )</td>
<td>( \text{segm} : \text{Train} \rightarrow \text{Segment} )</td>
</tr>
<tr>
<td>( \text{req} : \text{Segment} \rightarrow \mathbb{Z} )</td>
<td>( \text{next} : \text{Train} \rightarrow \text{Train} )</td>
</tr>
<tr>
<td>( \text{alloc} : \text{Segment} \rightarrow \mathbb{Z} )</td>
<td>( \text{spd} : \text{Train} \rightarrow \mathbb{R} )</td>
</tr>
<tr>
<td></td>
<td>( \text{pos} : \text{Train} \rightarrow \mathbb{R} )</td>
</tr>
<tr>
<td></td>
<td>( \text{prev} : \text{Train} \rightarrow \text{Train} )</td>
</tr>
<tr>
<td></td>
<td>( \text{[Train on segment]} )</td>
</tr>
<tr>
<td></td>
<td>( \text{[Requested by train]} )</td>
</tr>
<tr>
<td></td>
<td>( \text{[Allocated by train]} )</td>
</tr>
<tr>
<td></td>
<td>( \text{[Train segment]} )</td>
</tr>
<tr>
<td></td>
<td>( \text{[Next train]} )</td>
</tr>
<tr>
<td></td>
<td>( \text{[Speed]} )</td>
</tr>
<tr>
<td></td>
<td>( \text{[Current position]} )</td>
</tr>
<tr>
<td></td>
<td>( \text{[Prev. train]} )</td>
</tr>
</tbody>
</table>
```

- **13**

![Diagram](image-url)
Example: Controller for line track (RBC)

**OZ part.** Consists of data classes, axioms, the Init schema, update rules.

- 1. **Data classes** declare function symbols that can change their values during runs of the system, and are used in the OZ part of the specification.

- 2. **Axioms:** define properties of the data structures and system parameters which do not change
  - *gmax*: $\mathbb{R}$ (the global maximum speed),
  - *decmx*: $\mathbb{R}$ (the maximum deceleration of trains),
  - *d*: $\mathbb{R}$ (a safety distance between trains),
  - Properties of the data structures used to model trains/segments
Example: Controller for line track (RBC)

OZ part. Consists of data classes, axioms, the Init schema, update rules.

- 3. Init schema. describes the initial state of the system.
  - trains - doubly-linked list; placed correctly on the track segments
  - all trains respect their speed limits.

- 4. Update rules specify updates of the state space executed when the corresponding event from the CSP part is performed.

Example: Speed update

\[
\begin{align*}
\text{effect}_{\text{updSpd}} & \quad \forall t: \text{Train} \mid pos(t) < \text{length}(\text{segm}(t)) - d \land spd(t) - \text{decmx} \cdot \Delta t > 0 \\
& \quad \Gamma \max \{0, spd(t) - \text{decmx} \cdot \Delta t\} \leq spd'(t) \leq l\text{max}(\text{segm}(t)) \\
\text{effect}_{\text{updSpd}} & \quad \forall t: \text{Train} \mid pos(t) \geq \text{length}(\text{segm}(t)) - d \land \neg \text{alloc}(\text{nexts}(\text{segm}(t))) = tid(t) \\
& \quad \Gamma \max \{0, spd(t) - \text{decmx} \cdot \Delta t\} \leq spd'(t) \leq \min \{l\text{max}(\text{segm}(t)), l\text{max}(\text{nexts}(\text{segm}(t)))\} \\
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& \quad \Gamma spd'(t) = \max \{0, spd(t) - \text{decmx} \cdot \Delta t\}
\end{align*}
\]
Timed train controller (Train)

Train consists of three timed components running in parallel.

1. Update the train’s position.
   This component contains DC formulae of the form:
   \[
   \neg (true ; \downarrow updPos ; (\ell < \Delta t) ; \uparrow updPos ; true),
   \]
   \[
   \neg (true ; \downarrow updPos ; (\ell > c ) ; \uparrow updPos ; true),
   \]
   that specify lower/upper time bounds on \( updPos \) events.

2. Check if train is beyond the safety distance to the end of the segment.
   If so, it starts braking within a short reaction time.

3. Request extension of the movement authority from the RBC
   (may be granted or rejected).
Interaction RBC/Train

- **Train**
  - `1` updPos
  - `1` updSpd

- **RBC**
  - `1` req
  - `1` grant
  - `1` reject
  - `1` updPos
  - `1` updSpd

- **Environment**

- **TrainData**

- **SegmentData**
Overview

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- Modular Verification
- Modularity at structural level
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Modular Verification

<table>
<thead>
<tr>
<th>COD</th>
<th>Σ_S signature of S; T_S theory of S; T_S transition constraint system</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>specification</td>
</tr>
<tr>
<td></td>
<td>Init(\bar{x}); Update(\bar{x}, \bar{x}')</td>
</tr>
</tbody>
</table>

Given: Safe(x) formula (e.g. safety property)

- Invariant checking
  1. \models_{T_S} Init(\bar{x}) \rightarrow Safe(\bar{x})
     (Safe holds in the initial state)
  2. \models_{T_S} Safe(\bar{x}) \land Update(\bar{x}, \bar{x}') \rightarrow Safe(\bar{x}')
     (Safe holds before \Rightarrow holds after update)

- Bounded model checking (BMC):

  Check whether, for a fixed k, unsafe states are reachable in at most k steps, i.e. for all \(0 \leq j \leq k\):
  
  \[Init(x_0) \land Update_1(x_0, x_1) \land \cdots \land Update_n(x_{j-1}, x_j) \land \neg Safe(x_j) \models_{T_S} \bot\]
**Example 1: Speed Update**

\[
\begin{align*}
\text{pos}(t) &< \text{length}(\text{segm}(t)) - d \quad \rightarrow \quad 0 \leq \text{spd}'(t) \leq \text{lmax}(\text{segm}(t)) \\
\text{pos}(t) &\geq \text{length}(\text{segm}(t)) - d \quad \wedge \quad \text{alloc}(\text{next}_s(\text{segm}(t))) = \text{tid}(t) \\
\text{pos}(t) &\geq \text{length}(\text{segm}(t)) - d \quad \wedge \quad \text{alloc}(\text{next}_s(\text{segm}(t))) \neq \text{tid}(t) \\
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&\rightarrow \quad \text{spd}'(t) = \max(\text{spd}(t) - \text{decmax}, 0)
\end{align*}
\]
Trains on a linear track

Example 1: Speed Update

\[
\begin{align*}
pos(t) < \text{length}(\text{segm}(t)) - d & \rightarrow 0 \leq \text{spd}'(t) \leq \text{lmax}(\text{segm}(t)) \\
pos(t) \geq \text{length}(\text{segm}(t)) - d & \land \text{alloc}(\text{next}_s(\text{segm}(t))) = \text{tid}(t) \\
& \rightarrow 0 \leq \text{spd}'(t) \leq \min(\text{lmax}(\text{segm}(t)), \text{lmax}(\text{next}_s(\text{segm}(t)))) \\
pos(t) \geq \text{length}(\text{segm}(t)) - d & \land \text{alloc}(\text{next}_s(\text{segm}(t))) \neq \text{tid}(t) \\
& \rightarrow \text{spd}'(t) = \max(\text{spd}(t) - \text{decmax}, 0)
\end{align*}
\]

Proof task:

\[
\text{Safe}(\text{pos, next, prev, spd}) \land \text{SpeedUpdate}(\text{pos, next, prev, spd, spd}') \rightarrow \text{Safe}(\text{pos}', \text{next, prev, spd}')
\]
Incoming and outgoing trains

**Example 2:** Enter Update (also updates for segm’, spd’, pos’, train’)

**Assume:** \( s_1 \neq \text{null}_s, t_1 \neq \text{null}_t, \text{train}(s) \neq t_1, \text{alloc}(s_1) = \text{idt}(t_1) \)

\[
t \neq t_1, \text{ids}(\text{segm}(t)) < \text{ids}(s_1), \text{next}_t(t) = \text{null}_t, \text{alloc}(s_1) = \text{tid}(t_1) \rightarrow \text{next}’(t) = t_1 \land \text{next}’(t_1) = \text{null}_t
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→ $\text{next}'(t) = \text{next}_t(t)$

...

$t \neq t_1$, $\text{ids}(\text{segm}(t)) \geq \text{ids}(s_1)$ → $\text{next}'(t) = \text{next}_t(t)$
Safety property

Safety property we want to prove:
no two different trains ever occupy the same track segment:

(Safe) \( \forall t_1, t_2 \ \text{segm}(t_1) = \text{segm}(t_2) \rightarrow t_1 = t_2 \)

In order to prove that (Safe) is an invariant of the system, we need to find a suitable invariant \((\text{Inv}_i)\) for every control location \(i\) of the TCS, and prove:

(1) \((\text{Inv}_i)\) \models (Safe) for all locations \(i\) and

(2) the invariants are preserved under all transitions of the system,
\[(\text{Inv}_i) \land (\text{Update}) \models (\text{Inv}_j')\]
whenever \((\text{Update})\) is a transition from location \(i\) to \(j\).
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(Inv_i) \wedge (Update) \models (Inv_j')

whenever (Update) is a transition from location i to j.

Here: Inv_i generated by hand (use poss. of generating counterexamples with H-PILoT)
Verification problems

(1) \((\text{Inv}_i) \models (\text{Safe})\) for all locations \(i\) and

(2) the invariants are preserved under all transitions of the system,
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whenever (Update) is a transition from location \(i\) to \(j\).

Ground satisfiability problems for pointer data structures

**Problem:** Axioms, Invariants: are universally quantified

**Our solution:** Hierarchical reasoning in local theory extensions
Examples of theories we need to handle

- **Invariants**

  \((\text{Inv}_1)\) \(\forall t : \text{Train. } pc \neq \text{InitState} \land alloc(\text{next}_s(\text{segm}(t))) \neq \text{tid}(t)\)
  \(\rightarrow \text{length}(\text{segm}(t)) - \text{bd}(\text{spd}(t)) > \text{pos}(t) + \text{spd}(t) \cdot \Delta t\)

  \((\text{Inv}_2)\) \(\forall t : \text{Train. } pc \neq \text{InitState} \land \text{pos}(t) \geq \text{length}(\text{segm}(t)) - \text{d}\)
  \(\rightarrow \text{spd}(t) \leq \text{lmax}(\text{next}_s(\text{segm}(t)))\)
Examples of theories we need to handle

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  \[\rightarrow \text{spd}(t) \leq \text{lmax}(\text{next}_s(\text{segm}(t)))\]

- **Update rules**

  \[\forall t : \phi_1(t) \rightarrow s_1 \leq \text{spd}'(t) \leq t_1\]

  \[\ldots\]

  \[\forall t : \phi_n(t) \rightarrow s_n \leq \text{spd}'(t) \leq t_n\]
Modularity in automated reasoning

Examples of theories we need to handle

- **Invariants**
  
  \( (\text{Inv}_1) \forall t : \text{Train}. \ pc \neq \text{InitState} \land \text{alloc}(\text{next}_s(\text{segm}(t))) \neq \text{tid}(t) \)
  
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- **Update rules**

  \( \forall t : \phi_1(t) \rightarrow s_1 \leq \text{spd}'(t) \leq t_1 \)

  \[ \ldots \]

  \( \forall t : \phi_n(t) \rightarrow s_n \leq \text{spd}'(t) \leq t_n \)

- **Underlying theory:** theory of many-sorted pointers, real numbers, ...
Local theory extensions

Our approach: Find complete instantiations of univ. quantified variables

[VS’05] $\Sigma_0 \subseteq \Sigma \cup \Sigma$; $K$ clauses axiomatizing functions in $\Sigma$; $T_0$ $\Sigma_0$-theory;

\[ (\text{Loc}) \quad T_0 \subseteq T_1 = T_0 \cup K \text{ is local, if for any (finite) set of ground clauses } G, \]
\[ T_0 \cup K \cup G \models \bot \quad \text{iff} \quad T_0 \cup K[G] \cup G \models \bot \]
\[ \Leftarrow \quad \text{always} \]
\[ \Rightarrow \quad \text{locality} \]

Various notions of locality, depending of the instances to be considered closure operator on ground terms: [Ihlemann, Jacobs, VS’08, Ihlemann, VS’10]
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& \mathcal{T}_0 \cup \mathcal{K} \cup G \models \bot \quad \text{iff} \quad \mathcal{T}_0 \cup \mathcal{K}[G] \cup G \models \bot \\
& \Leftarrow \quad \text{always} \\
& \Rightarrow \quad \text{locality}
\end{align*}

Various notions of locality, depending of the instances to be considered 
closure operator on ground terms: [Ihlemann, Jacobs, VS’08, Ihlemann, VS’10]

Main advantages: $\mapsto$ hierarchical reduction to proof tasks in $\mathcal{T}_0$ 
$\mapsto$ decision procedure for satisfiability of ground clauses 
$\mapsto$ implementation H-PILoT [Ihlemann, VS’2009]
Example: doubly-linked lists

\[ \forall p \ (p \neq \text{null} \land p.\text{next} \neq \text{null} \rightarrow p.\text{next} . \text{prev} = p) \]

\[ \forall p \ (p \neq \text{null} \land p.\text{prev} \neq \text{null} \rightarrow p.\text{prev} . \text{next} = p) \]

\[ \land \ c \neq \text{null} \land c.\text{next} \neq \text{null} \land d \neq \text{null} \land d.\text{next} \neq \text{null} \land c.\text{next} = d.\text{next} \land c \neq d \quad \models \quad \bot \]
Example: doubly-linked lists

\[(c \neq \text{null} \land c.\text{next} \neq \text{null} \rightarrow c.\text{next}.\text{prev} = c) \quad (c.\text{next} \neq \text{null} \land c.\text{next}.\text{next} = \text{null} \rightarrow c.\text{next}.\text{next}.\text{prev} = c.\text{next})\]
\[(d \neq \text{null} \land d.\text{next} \neq \text{null} \rightarrow d.\text{next}.\text{prev} = d) \quad (d.\text{next} \neq \text{null} \land d.\text{next}.\text{next} = \text{null} \rightarrow d.\text{next}.\text{next}.\text{prev} = d.\text{next})\]
\[\land \quad c \neq \text{null} \land c.\text{next} \neq \text{null} \land d \neq \text{null} \land d.\text{next} \neq \text{null} \land c.\text{next} = d.\text{next} \land c \neq d \quad \models \quad \bot\]

Similar results also if numerical info is stored in list
The good news

The following sets of formulae define local theory extensions:

- Updates (according to a partition of the state space)
- The invariants we consider
- The axioms for many-sorted pointer structures we consider
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To show:

$\mathcal{T}_2 = \mathcal{T}_1 \cup \text{Update} (\text{next}, \ldots, \text{next}', \ldots)$

$\mathcal{T}_1 = \mathcal{T}_0 \cup \text{Inv} (\text{next}, \ldots)$

$\mathcal{T}_0 = (\text{Pointers, } \mathbb{R})$

$\mathcal{T}_2 \cup \neg \text{Inv} (\text{next}') \models \bot$
The good news

The following sets of formulae define local theory extensions:

- Updates (according to a partition of the state space)
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\[\mathcal{T}_2 = \mathcal{T}_1 \cup \text{Update}(\text{next}, ... \text{next}', ...)\]
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\[\mathcal{T}_0 = (\text{Pointers}, \mathbb{R})\]

To show:

\[\mathcal{T}_2 \cup \neg \text{Inv}(\text{next}') \models \bot\]
\[\mathcal{T}_1 \cup \text{Update}[G] \land G \models \bot\]
\[\mathcal{T}_0 \cup \text{Inv}[G'] \land G' \models \bot\]
\[UIF \cup \mathbb{R} \cup (\text{PointerAx}[G''] \cup G'')_0 \models \bot\]

H-PILoT: verification/models/QE \xrightarrow{\text{}} constraints on parameters
Overview

• Modular Specifications: CSP-OZ-DC

• Modular Verification

• Modularity at structural level

• Implementation; experimental results

• Conclusions
Modularity at structural level

- Complex track topologies

Assumptions:
- No cycles
- in-degree (out-degree) of associated graph at most 2.

Approach:
- Decompose the system in trajectories (linear rail tracks; may overlap)
- **Task 1**: Prove safety for trajectories with incoming/outgoing trains
  - Conclude that for control rules in which trains have sufficient freedom (and if trains are assigned unique priorities) safety of all trajectories implies safety of the whole system
- **Task 2**: General constraints on parameters which guarantee safety
Overview

- Modular Specifications: CSP-OZ-DC
- Modular Verification
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- Implementation; experimental results
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Tool Chain

Syspect

UML → CSP-OZ-DC

PEA

PEA toolkit

TCS

ARMC

H-PILoT

TCS

Prover
Experimental results

Verification of RBC

<table>
<thead>
<tr>
<th>Verification of RBC</th>
<th>(Syspect + PEA)</th>
<th>(H-PILoT + Yices)</th>
<th>(Yices alone)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Inv) <em>unsat</em></td>
<td>11s</td>
<td>72s</td>
<td>52s</td>
</tr>
<tr>
<td>Part 1</td>
<td>11s</td>
<td>124s</td>
<td>131s</td>
</tr>
<tr>
<td>Part 2</td>
<td>11s</td>
<td>8s</td>
<td>45s</td>
</tr>
<tr>
<td>speed update</td>
<td>11s</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Safe) <em>sat</em></td>
<td>9s</td>
<td>8s (+ model)</td>
<td>time out</td>
</tr>
<tr>
<td>Consistency</td>
<td>13s</td>
<td>3s</td>
<td>(Unknown) 2s</td>
</tr>
</tbody>
</table>

(obtained on: AMD64, dual-core 2 GHz, 4 GB RAM)

Verification of **Train**: 8 parallel components, $>3300$ transitions, 28 real-valued variables, clocks (infinite state system).
For this reason, the verification took 26 hours
Summary

Main approach: Exploit modularity in specification/verification/structure

Contributions: [Faber, Ihlemann, Jacobs, VS, 2010]

- We augmented existing techniques for the verification of real-time systems to cope with rich data structures like pointer structures (and identified a decidable fragment of this theory).
- We established various modularity results.
- We implemented our approach in a new tool chain taking high-level specifications in terms of COD as input.
Beyond Yes/No

We consider parametric systems
– parametric data, parametric change, parametric environment (functions)
– parametric topology of the system (data structures)

**Given:** Safety property (formula $\Phi$)

**Task:**
1. Check if constraints on parameters guarantee safety
2. Infer relationships between parameters, resp. properties of the functions modeling the changes which ensure that the safety property $\Phi$ is an invariant
3. Find models (situations when safety property does not hold)

[VS; IJCAR’10] and [VS: CADE’13]

- Use the “good” properties of theories occurring in verification
- Exploit possibilities for
  ‘ hierarchical reasoning (1), quantifier elimination (2), model building (3)
Further extensions

[Damm, Horbach, VS: FroCoS’15] Modularity results and small model property results for (decoupled) families of linear hybrid automata

Examples:

Sensors + Communication Channels

**Safety properties:** \( \forall i_1, \ldots, i_k \quad \phi_{\text{safe}}(i_1, \ldots, i_k) \)

Collision free: \( \forall i, j(\text{lng}(i) = \text{lng}(j) \land \text{pos}(i) \geq \text{pos}(j) \land i \neq j \rightarrow \text{pos}(i) - \text{pos}(j) > d) \)
Conclusions

Main approach: Exploit modularity in specification/verification/structure

Application areas:
- Verification of real time systems [Faber, Ihlemann, Jacobs, VS’10]
- Verification of hybrid systems [Damm, Horbach, VS’15]

Main idea:
- Use locality of the decidable fragment of the theory of pointers and of updates to simplify verification tasks.
- By-product: Small model property, complexity estimation
- Parametric verification and model building possible

Implementations
- Chain tool for real time systems
- Verification tool for families of LHA
Conclusions

Main approach: Exploit modularity in specification/verification/structure

Application areas:

- Verification of real time systems [Faber, Ihlemann, Jacobs, VS’10]
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Main idea:

- Use locality of the decidable fragment of the theory of pointers and of updates to simplify verification tasks.
- By-product: Small model property, complexity estimation
- Parametric verification and model building possible

Ongoing and future work: More complex combinations/properties
  - Time-bounded reachability conditions (e.g. overtaking manoeuvres)
  - Invariant generation