On Distribution Based Bisimulations for Probabilistic Automata

AVACS alumni technical talk

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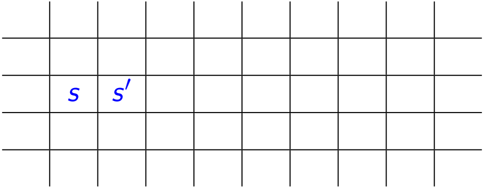
29th September, 2015

Motivation: why bisimulations?

Behavioral Equivalences: Bisimulations, simulations, ...

- Bisimulation is a key notion in system modelling and verification to reduce the state space, etc.



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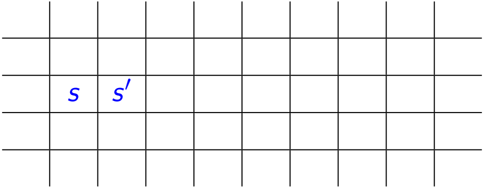

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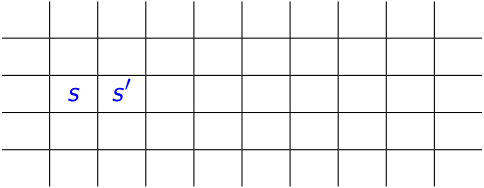


- from the 926 AVACS publications, many of them are about simulations/bisimulations...

- Yesterday, today and tomorrow ...

Yesterday: my time at Saarland

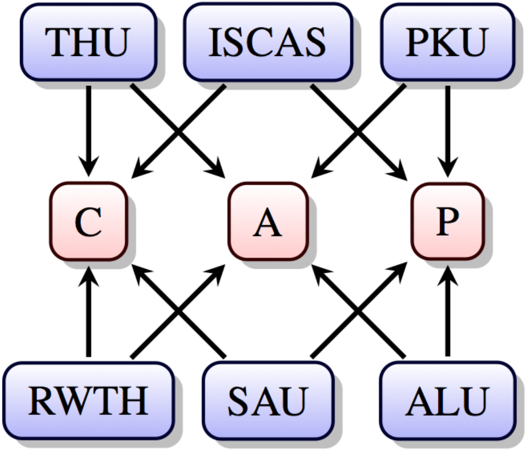
My dissertation is about deciding simulations for probabilistic automata..., based on...

- Lijun Zhang, Holger Hermanns, Friedrich Eisenbrand, and David N. Jansen. Flow Faster: Efficient Decision Algorithms for Probabilistic Simulations. In Tools and Algorithms for the Construction and Analysis of Systems, 13th International Conference (TACAS), volume 4424 of LNCS, pages 155-169. Springer-Verlag, 2007.
- Lijun Zhang and Holger Hermanns. Deciding Simulations on Probabilistic Automata. In Automated Technology for Verification and Analysis, 5th International Symposium (ATVA), volume 4762 of LNCS, pages 207-222. Springer-Verlag, 2007.
- Lijun Zhang. A Space-Efficient Probabilistic Simulation Algorithm. In Concurrency Theory, 19th International Conference, (CONCUR), volume 5201 of LNCS, pages 248-263. Springer-Verlag, 2008.
- Lijun Zhang, Holger Hermanns, Friedrich Eisenbrand, and David N. Jansen. Flow Faster: Efficient Decision Algorithms for Probabilistic Simulations. Logical Methods in Computer Science, 4(4), 2008.
- Holger Hermanns, Björn Wachter, and Lijun Zhang. Probabilistic CEGAR. In Computer Aided Verification, 20th International Conference (CAV), volume 5123 of LNCS, pages 162-175. Springer-Verlag, 2008.
- Lijun Zhang, Zhikun She, Stefan Ratschan, Holger Hermanns, and Ernst Moritz Hahn. Safety Verification for Probabilistic Hybrid Systems. In Computer Aided Verification, 22th International Conference (CAV), volume 6174 of LNCS, pages 196-211. Springer-Verlag, 2010.
- Ernst Moritz Hahn, Holger Hermanns, Björn Wachter, and Lijun Zhang. PARAM: A Model Checker for Parametric Markov Models. In Computer Aided Verification, 22th International Conference (CAV), volume 6174 of LNCS, pages 660-664. Springer-Verlag, 2010.
- Christian Eisentraut, Holger Hermanns, and Lijun Zhang. On Probabilistic Automata in Continuous Time. In 25th Annual IEEE Symposium on Logic in Computer Science (LICS), pages 342-351. IEEE CS Press, 2010.

Today: my time at Oxford/DTU/ISCAS

- Lei Song, Lijun Zhang, and Jens C. Godskesen. Bisimulations Meet PCTL Equivalences for Probabilistic Automata. In Concurrency Theory, 22nd International Conference (CONCUR), volume 6901 of LNCS, pages 108-123. Springer-Verlag, 2011.
- Holger Hermanns, Augusto Parma, Roberto Segala, Björn Wachter, and Lijun Zhang. Probabilistic Logical Characterization. Information and Computation, 209(2):154-172, 2011.
- Christian Eisentraut, Holger Hermanns, Johann Schuster, Andrea Turrini, and Lijun Zhang. The Quest for Minimal Quotients for Probabilistic Automata. In Tools and Algorithms for the Construction and Analysis of Systems, 19th International Conference (TACAS), volume 7795 of LNCS, pages 16-31. Springer-Verlag, 2013.
- Lei Song, Lijun Zhang, Jens C. Godskesen, and Flemming Nielson. Bisimulations Meet PCTL Equivalences for Probabilistic Automata. Logical Methods in Computer Science, 9(2), 2013.
- Yuan Feng and Lijun Zhang. When Equivalence and Bisimulation Join Forces in Probabilistic Automata. In Nineteenth international symposium of the Formal Methods Europe association (FM), volume 8442 of LNCS, pages 247-262. Springer, 2014.
- Lei Song, Lijun Zhang, Holger Hermanns, and Jens Chr. Godskesen. Incremental Bisimulation Abstraction Refinement. Transactions on Embedded Computing Systems (TECS), 13(4s):142:1-142:23, 2014.
- Christian Eisentraut, Jens Chr. Godskesen, Holger Hermanns, Lei Song, and Lijun Zhang. Probabilistic bisimulation for realistic schedulers. In Formal Methods - 20th International Symposium (FM), volume 9109 of LNCS, pages 248-264. Springer, 2015.
- Lei Song, Yuan Feng, and Lijun Zhang. Distributed bisimulation for multi-agent systems. In International Conference on Autonomous Agents & Multiagent Systems (AAMAS), pages 209-217. ACM, 2015.
- Fei He, Xiaowei Gao, Bow-Yaw Wang, and Lijun Zhang. Leveraging weighted automata in compositional reasoning about concurrent probabilistic systems. In 42nd ACM SIGPLAN-SIGACT Symposium on Principles of Programming Languages (POPL), pages 503-514. ACM, 2015.

Tomorrow... CAP



Message of this talk

State based bisimulations for probabilistic systems have too powerful distinguishing power: we need distributions!

Outline

- Motivation
- Bisimulations for Labelled Transition Systems
- State & Distribution based Bisimulation for Probabilistic Automata
- Weak BIsimulation for Probabilistic Automata
- Conclusion

Labelled Transition System

A labelled transition system (LTS) is a tuple $\mathcal{A} = (S, Act, \rightarrow)$ where

- S is a finite set of states,
- Act is a finite set of actions,
- $\rightarrow \subseteq S \times Act \times S$ is a transition relation.

We write $s \xrightarrow{a} s'$ if $(s, a, s') \in \rightarrow$.

Bisimulation for LTS

Given a LTS $\mathcal{A} = (S, Act, \to)$, a binary relation $\mathcal{R} \subseteq S \times S$ is a bisimulation if $s\mathcal{R}t$ implies that

1. $\forall s \xrightarrow{a} s'$, $\exists t \xrightarrow{a} t'$ such that $s'\mathcal{R}t'$, and
2. symmetrically, $\forall t \xrightarrow{a} t'$, $\exists s \xrightarrow{a} s'$ such that $s'\mathcal{R}t'$,.

We write $s \sim t$ whenever there is a bisimulation $\mathcal{R}$ such that $s\mathcal{R}t$.

Simulation for LTS

Given a LTS $\mathcal{A} = (S, Act, \rightarrow)$, a binary relation $\mathcal{R} \subseteq S \times S$ is a simulation if $s\mathcal{R}t$ implies that

1. $\forall s \xrightarrow{a} s'$, $\exists t \xrightarrow{a} t'$ such that $s'\mathcal{R}t'$.

We write $s \precsim t$ whenever there is a bisimulation $\mathcal{R}$ such that $s\mathcal{R}t$.

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Some properties:

1. $\sim$ is an equivalence relation, its the largest bisimulation relation.
2. $\precsim$ is a preorder, its the largest simulation relation.

Bisimulation for LTS

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bisimulation is not necessarily equivalence relation!

Probabilistic automata

Let $\mathcal{D}(S)$ denote the set of distributions over S. A probabilistic automaton (PA) is a tuple $\mathcal{A} = (S, Act, \rightarrow)$ where

- S is a finite set of states,
- Act is a finite set of actions,
- $\rightarrow \subseteq S \times Act \times \mathcal{D}(S)$ is a probabilistic transition relation.

We write $s \xrightarrow{a} \mu$ if $(s, a, \mu) \in \rightarrow$.

1. A LTS is a PA with only Dirac distributions.
2. A Markov chain is a PA such that $s \xrightarrow{a} \mu$ and $s \xrightarrow{a'} \mu'$ imply $a = a', \mu = \mu'$.

Bisimulation for LTS: how to extend it for PAs?

Given a LTS $\mathcal{A} = (S, Act, \rightarrow)$, a binary relation $\mathcal{R} \subseteq S \times S$ is a bisimulation if $s\mathcal{R}t$ implies that

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Adapt the first condition by:
$\forall s \xrightarrow{a} \mu$, $\exists t \xrightarrow{a} \mu'$ such that $\mu\mathcal{R}\mu'$

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how to define $\mu\mathcal{R}\mu'$?

(State-based) probabilistic bisimulation   Larsen & Skou'89

Lifted relation: looks simpler, but more restrict for defining simulations

Given a PA $\mathcal{A} = (S, Act, \rightarrow)$, an equivalence relation $\mathcal{R} \subseteq S \times S$ is a bisimulation if $s\mathcal{R}t$ implies that

- $\forall s \xrightarrow{a} \mu$, $\exists t \xrightarrow{a}_{\mathsf{P}} \nu$ such that for all equivalence class $C \in S/\mathcal{R}$, $\mu(C) = \nu(C)$.

We write $s \sim t$ whenever there is a bisimulation $\mathcal{R}$ such that $s\mathcal{R}t$.

(State-based) probabilistic simulation   Jonsson & Larsen 91

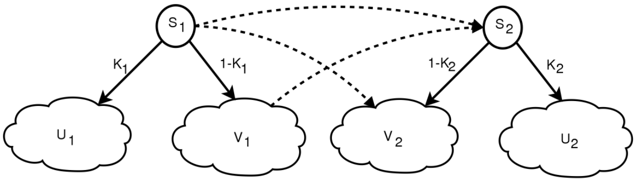
Lifted relation: weight functions are used for defining simulations

Let $\mathcal{R} \subseteq S \times S$, and μ and ν be two distributions. Then $\mu\ \mathcal{R}^\dagger\ \nu$ if there exists a weight function $w : S \times S \to [0,1]$ such that

1. $\forall s : \sum_{t \in S} w(s,t) = \mu(s)$
2. $\forall t : \sum_{s \in S} w(s,t) = \nu(t)$
3. $\forall (s,t) : w(s,t) > 0 \Rightarrow s\mathcal{R}t$.

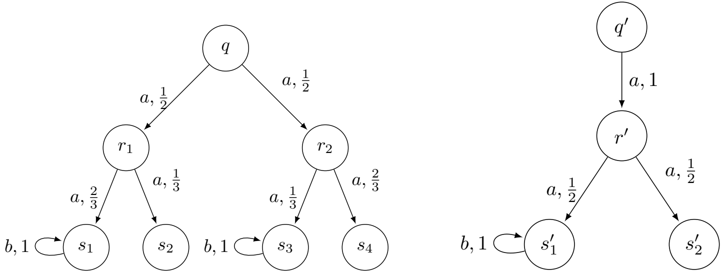
Summary

bisimulations with lifting are complex



Baier et. al.

Distinguishing power of state-based bisimulation

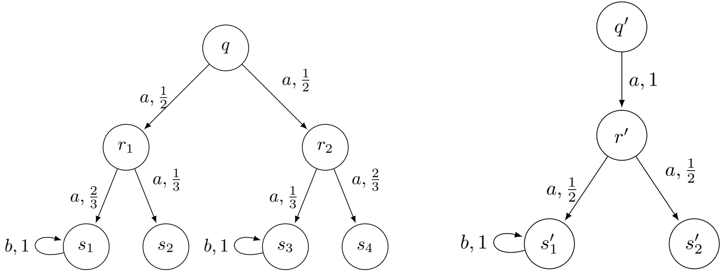


Obviously, $r_1 \not\sim r'$ and $r_2 \not\sim r'$. Thus

$$\frac{1}{2}r_1 + \frac{1}{2}r_2 \not\sim \bar{r'}$$

So $q \not\sim q'$!

Distinguishing power of state-based bisimulation



However, the states q and q' should be bisimilar, since it should holds

$$\frac{1}{2}r_1 + \frac{1}{2}r_2 \sim \bar{r'}.$$

Key observation: we need to define $\sim$ directly over $\mathcal{D}(S)$, not a lifted relation from S!.

Solution: Distribution-based bisimulation Doyen et. al.'08

For labelled Markov chains

- A relation $\mathcal{R} \subseteq \mathcal{D}(S) \times \mathcal{D}(S)$ is a bisimulation if $\mu \mathcal{R} \nu$ implies that
  - $\mu(F) = \nu(F)$;
  - $(M_\alpha \mu)\ \mathcal{R}\ (M_\alpha \nu)$ for all $\alpha \in Act$.

Distribution-based bisimulation for PA

A relation $\mathcal{R} \subseteq \mathcal{D}(S) \times \mathcal{D}(S)$ is a bisimulation if $\mu \mathcal{R} \nu$ implies that

1. $\forall \mu \xrightarrow{a} \mu'$, $\exists \nu \xrightarrow{a} \nu'$ such that $\mu' \mathcal{R} \nu'$.
2. symmetrically for ν.

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- But, how to define $\mu \xrightarrow{a} \mu'$?
- Answer: Lifted transitions!

Lifted transitions

- A natural definition: $\mu \xrightarrow{a} \mu'$ if

$$\forall s \in \lceil \mu \rceil, \exists s \xrightarrow{a}_{\mathrm{P}} \mu_s \text{ such that } \mu' = \sum_s \mu(s) \cdot \mu_s.$$

- Only definable for input enabled systems; that is, $Act(s) = Act$ for all $s \in S$, where $Act(s) := \{a \mid s \xrightarrow{a} \mu\}$ is the set of enabled actions in s.

Lifted transitions

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- Only definable for input enabled systems; that is, $Act(s) = Act$ for all $s \in S$, where $Act(s) := \{a \mid s \xrightarrow{a} \mu\}$ is the set of enabled actions in s.
- How about general PA? Input enabled consistent splitting!

Distribution-based bisimulation for PA

A distribution μ is transition consistent, written as $\overrightarrow{\mu}$, if all states in its support have the same set of enabled actions.

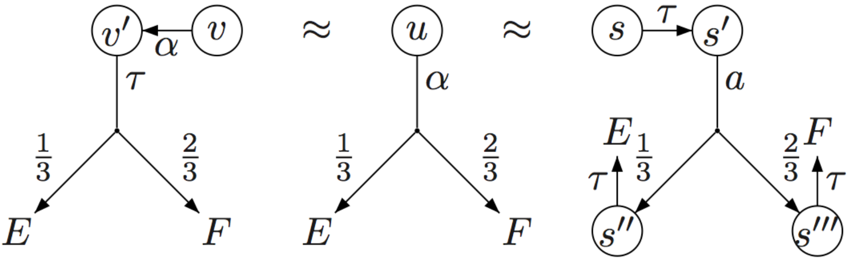
Definition

A relation $\mathcal{R} \subseteq \mathit{Dist}(S) \times \mathit{Dist}(S)$ is a *distribution-based bisimulation* iff $\mu \; \mathcal{R} \; \nu$ implies:

1. $\forall \mu \xrightarrow{a} \mu'$, $\exists \nu \xrightarrow{a} \nu'$ such that $\mu' \mathcal{R} \nu'$.
2. if not $\overrightarrow{\mu}$, then there exists $\mu \equiv \sum_{0 \leq i \leq n} p_i \cdot \mu_i$ and $\nu \equiv \sum_{0 \leq i \leq n} p_i \cdot \nu_i$ such that $\overrightarrow{\mu_i}$ and $\mu_i \; \mathcal{R} \; \nu_i$ for each $0 \leq i \leq n$ where $\sum_{0 \leq i \leq n} p_i = 1$ with $p_i > 0$ for each i.
3. symmetrically for ν.

We say that μ and ν are distribution-based bisimilar iff there exists a distribution-based bisimulation $\mathcal{R}$ with $\mu \mathcal{R} \nu$.

Weak Bisimulation Yardsticks



Weak Transitions

- Define $\mu \xrightarrow{\alpha} \mu'$ iff there exists a transition $s \xrightarrow{\alpha} \mu_s$ for each $s \in Supp(\mu)$ such that $\mu' = \sum_{s \in Supp(\mu)} \mu(s) \cdot \mu_s$.
- Then, $s \stackrel{\tau}{\Longrightarrow} \mu$ iff there exists

$$\begin{array}{ccc} \mathcal{D}_s & = & \mu_0^{\rightarrow} + \mu_0^{\times}, \\ \mu_0^{\rightarrow} & \xrightarrow{\tau} & \mu_1^{\rightarrow} + \mu_1^{\times}, \\ \mu_1^{\rightarrow} & \xrightarrow{\tau} & \mu_2^{\rightarrow} + \mu_2^{\times}, \\ & \cdots & \end{array}$$

  where $\mu = \sum_{i \geq 0} \mu_i^{\times}$. We write $s \stackrel{\alpha}{\Longrightarrow} \mu$ iff there exists $s \stackrel{\tau}{\Longrightarrow} \xrightarrow{\alpha} \stackrel{\tau}{\Longrightarrow} \mu$.
- $\stackrel{a}{\Longrightarrow}_C$ is defined similarly.

(State-based) Weak Bisimulation

Definition

An equivalence relation $\mathcal{R} \subseteq S \times S$ is a state-based bisimulation iff $s \ \mathcal{R} \ r$ implies that

- for all $s \xrightarrow{a} \mu$, there exists a weak transition $r \overset{a}{\Longrightarrow}_{\mathrm{C}} \mu'$ such that for all equivalence class C: $\mu(C) = \mu'(C)$.

Weak Bisimulation

LICS'10

Definition 11 (Weak Bisimulation). *A relation $\mathcal{R}$ over subdistributions over S is called a weak bisimulation if whenever $\mu_1 \; \mathcal{R} \; \mu_2$ then for all $\alpha \in Act^\chi$:*

$$|\mu_1| = |\mu_2| \qquad (1)$$

and

A.) $\forall E \in Supp(\mu_1)$: $\exists {\mu_2}^g, {\mu_2}^s$: $\mu_2 \Longrightarrow_C {\mu_2}^g \oplus {\mu_2}^s$ *and*

- *(i)* $[\![(E, \mu_1(E))]\!] \; \mathcal{R} \; {\mu_2}^g$ *and* $(\mu_1 - E) \; \mathcal{R} \; {\mu_2}^s$
- *(ii) whenever* $E \xrightarrow{\alpha} \mu_1'$ *for some* μ_1' *then* ${\mu_2}^g \stackrel{\hat{\alpha}}{\Longrightarrow}_C \mu''$ *and* $\mu_1(E) \cdot \mu_1' \; \mathcal{R} \; \mu''$

and

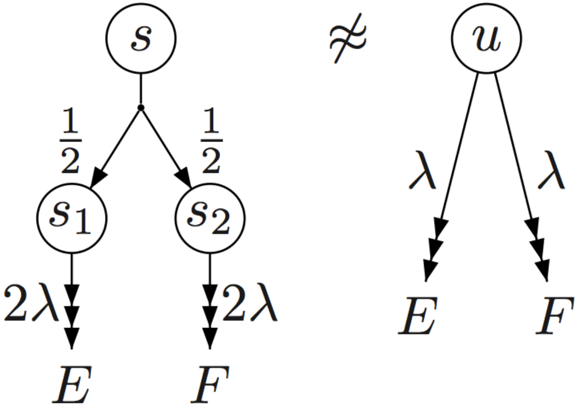
B.) $\forall F \in Supp(\mu_2)$: $\exists {\mu_1}^g, {\mu_1}^s$: $\mu_1 \Longrightarrow_C {\mu_1}^g \oplus {\mu_1}^s$ *and*

- *(i)* ${\mu_1}^g \; \mathcal{R} \; [\![(F, \mu_2(F))]\!]$ *and* ${\mu_1}^s \; \mathcal{R} \; (\mu_2 - F)$
- *(ii) whenever* $F \xrightarrow{\alpha} \mu_2'$ *for some* μ_2' *then* ${\mu_1}^g \stackrel{\hat{\alpha}}{\Longrightarrow}_C \mu''$ *and* $\mu'' \; \mathcal{R} \; \mu_2(F) \cdot \mu_2'$

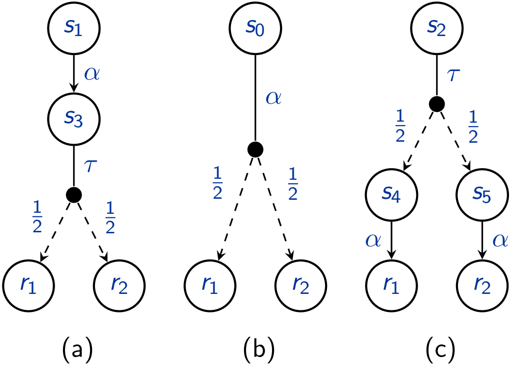
LICS Open Problem

LICS'10

Finally, we remark that the quest for a good notion of equality is tightly linked to the practically relevant issue of constructing a small (quotient) model that contains all relevant information needed to analyse the system, or to compose it further.
From this perspective, there are still equalities that one may (or may not) consider desirable...



LICS Open Problem



Decomposability: Source of the matters

See clause B, we consider all F in the support individually:

(i) $[\![(E, \mu_1(E))]\!] \mathcal{R} {\mu_2}^g$ *and* $(\mu_1 - E) \mathcal{R} {\mu_2}^s$
(ii) *whenever* $E \xrightarrow{\alpha} \mu_1'$ *for some* μ_1' *then* ${\mu_2}^g \overset{\hat{\alpha}}{\Longrightarrow}_C \mu''$ *and* $\mu_1(E) \cdot \mu_1' \mathcal{R} \mu''$

and

B.) $\forall F \in Supp(\mu_2)$: $\exists {\mu_1}^g, {\mu_1}^s$: $\mu_1 \Longrightarrow_C {\mu_1}^g \oplus {\mu_1}^s$ *and*
(i) ${\mu_1}^g \mathcal{R} [\![(F, \mu_2(F))]\!]$ *and* ${\mu_1}^s \mathcal{R} (\mu_2 - F)$
(ii) *whenever* $F \xrightarrow{\alpha} \mu_2'$ *for some* μ_2' *then* ${\mu_1}^g \overset{\hat{\alpha}}{\Longrightarrow}_C \mu''$ *and* $\mu'' \mathcal{R} \mu_2(F) \cdot \mu_2'$

Late Weak Bisimulation

Definition

A distribution μ is transition consistent, written as $\overrightarrow{\mu}$, if for any $s \in Supp(\mu)$ and $\alpha \neq \tau$, $s \stackrel{\alpha}{\Longrightarrow} \gamma$ for some γ implies $\mu \stackrel{\alpha}{\Longrightarrow} \gamma'$ for some γ'.

For a distribution being transition consistent, all states in the support of the distribution should have the same set of enabled visible actions.

Late Weak Bisimulation

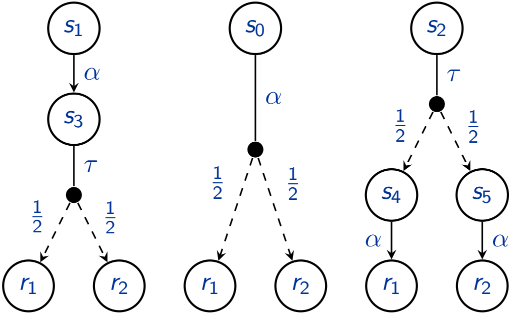
Definition

$\mathcal{R} \subseteq \mathit{Dist}(S) \times \mathit{Dist}(S)$ is a *late distribution bisimulation* iff $\mu \mathrel{\mathcal{R}} \nu$ implies:

1. whenever $\mu \xhookrightarrow{\alpha}_{\mathsf{C}} \mu'$, there exists a $\nu \xRightarrow{\alpha}_{\mathsf{C}} \nu'$ such that $\mu' \mathrel{\mathcal{R}} \nu'$;
2. if not $\overrightarrow{\mu}$, then there exists $\mu = \sum_{0 \le i \le n} p_i \cdot \mu_i$ and $\nu \xRightarrow{\tau}_{\mathsf{C}} \sum_{0 \le i \le n} p_i \cdot \nu_i$ such that $\overrightarrow{\mu_i}$ and $\mu_i \mathrel{\mathcal{R}} \nu_i$ for each $0 \le i \le n$ where $\sum_{0 \le i \le n} p_i = 1$;
3. symmetrically for ν.

We say that μ and ν are late distribution bisimilar, written as $\mu \approx^{\bullet} \nu$, iff there exists a late distribution bisimulation $\mathcal{R}$ such that $\mu \mathrel{\mathcal{R}} \nu$. Moreover $s \approx^{\bullet} r$ iff $\mathcal{D}_s \approx^{\bullet} \mathcal{D}_r$.

LICS Open Problem



(a) (b) (c)

Late Weak Bisimulation wrt Schedulers

A scheduler is a function from finite paths to distribution of *enabled transitions*.

Definition

Let $\xi_1, \xi_2, \xi \in \mathcal{S}$ for a given set of schedulers $\mathcal{S}$. $\mathcal{R} \subseteq \mathit{Dist}(S) \times \mathit{Dist}(S)$ is a late distribution bisimulation with respect to $\mathcal{S}$ iff $\mu \; \mathcal{R} \; \nu$ implies:

1. whenever $\mu \xrightarrow{\alpha}_{\xi_1} \mu'$, there exists $\nu \overset{\alpha}{\Longrightarrow}_{\xi_2} \nu'$ such that $\mu' \; \mathcal{R} \; \nu'$;
2. if not $\overrightarrow{\mu}$, then there exists $\mu = \sum_{0 \leq i \leq n} p_i \cdot \mu_i$ and $\nu \overset{\tau}{\Longrightarrow}_{\xi} \sum_{0 \leq i \leq n} p_i \cdot \nu_i$ such that $\overrightarrow{\mu_i}$ and $\mu_i \; \mathcal{R} \; \nu_i$ for each $0 \leq i \leq n$ where $\sum_{0 \leq i \leq n} p_i = 1$;
3. symmetrically for ν.

We write $\mu \overset{\bullet}{\approx}_{\mathcal{S}} \nu$ iff there exists a late distribution bisimulation $\mathcal{R}$ with respect to $\mathcal{S}$ such that $\mu \; \mathcal{R} \; \nu$. And we write $s \overset{\bullet}{\approx}_{\mathcal{S}} r$ iff $\mathcal{D}_s \overset{\bullet}{\approx}_{\mathcal{S}} \mathcal{D}_r$.

Late Weak Bisimulation wrt Schedulers

Realistic Schedulers

- Partial Information Schedulers (deAlfaro) $\mathcal{S}_P$: it can only distinguish states via different enabled visible actions.
- Distributed Schedulers (Giro & D'Argenio) $\mathcal{S}_D$: each component can use only that information about other components that has been conveyed to it beforehand

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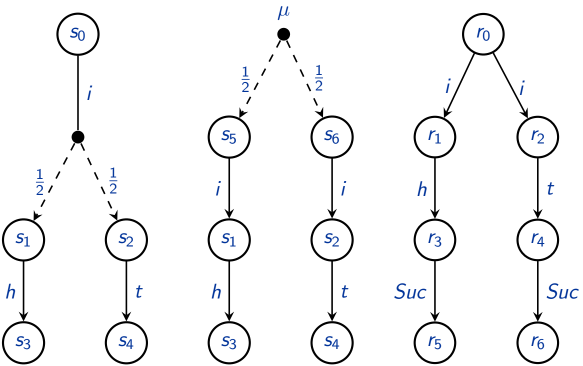

Late Weak Bisimulation wrt Schedulers

Definition

Let $\xi_1, \xi_2, \xi \in \mathcal{S}$ for a given set of schedulers $\mathcal{S}$. $\mathcal{R} \subseteq \mathit{Dist}(S) \times \mathit{Dist}(S)$ is a late distribution bisimulation with respect to $\mathcal{S}$ iff $\mu \ \mathcal{R} \ \nu$ implies:

1. whenever $\mu \xrightarrow{\alpha}_{\xi_1} \mu'$, there exists $\nu \overset{\alpha}{\Longrightarrow}_{\xi_2} \nu'$ such that $\mu' \ \mathcal{R} \ \nu'$;
2. if not $\overrightarrow{\mu}$, then there exists $\mu = \sum_{0 \le i \le n} p_i \cdot \mu_i$ and $\nu \overset{\tau}{\Longrightarrow}_{\xi} \sum_{0 \le i \le n} p_i \cdot \nu_i$ such that $\overrightarrow{\mu_i}$ and $\mu_i \ \mathcal{R} \ \nu_i$ for each $0 \le i \le n$ where $\sum_{0 \le i \le n} p_i = 1$;
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We write $\mu \ \overset{\bullet}{\approx}_{\mathcal{S}} \ \nu$ iff there exists a late distribution bisimulation $\mathcal{R}$ with respect to $\mathcal{S}$ such that $\mu \ \mathcal{R} \ \nu$. And we write $s \ \overset{\bullet}{\approx}_{\mathcal{S}} \ r$ iff $\mathcal{D}_s \ \overset{\bullet}{\approx}_{\mathcal{S}} \ \mathcal{D}_r$.

In the above definition, every transition is induced by a scheduler in $\mathcal{S}$. Obviously, when $\mathcal{S}$ is the set of all schedulers, these two definitions coincide. Thus, $s_1 \ \overset{\bullet}{\approx} \ s_2 \Longleftrightarrow s_1 \ \overset{\bullet}{\approx}_{\mathcal{S}_D} \ s_2$, provided s_1 and s_2 contain no parallel operators, as in this case $\mathcal{S}_D$ represents the set of all schedulers.

Late Weak Bisimulation wrt Schedulers

Theorem

For any states s_1 *and* s_2, $s_1 \approx^{\bullet} s_2$ *iff* $s_1 \approx^{\bullet}_{\mathcal{S}_P} s_2$.

Theorem

For any states s_1, s_2, *and* s_3,

$$s_1 \approx^{\bullet}_{\mathcal{S}_D} s_2 \text{ implies } s_1 \parallel_A s_3 \approx^{\bullet}_{\mathcal{S}_D} s_2 \parallel_A s_3.$$

Conclusions

Distribution Based Bisimulation

- Bisimulation based on distributions
- Coarser than existing weak bisimulation
- Nice properties wrt. realistic schedulers

More extensions and properties

- Markov automata
- Relation to trace distribution
- Decision algorithm

Related work

- Non-probabilistic models

  [Park & Milner]

- Strong (bi-)simulations, weak bisimulations for Markov chains, probabilistic automata

  [Jonsson & Larsen & Segala]

- On Probabilistic Automata in Continuous Time

  [Eisentraut, Hermanns & Zhang @ LICS'10]

- When Equivalence and Bisimulation Join Forces in Probabilistic Automata

  [Feng & Zhang @ FM'14]

- Probabilistic Bisimulation: Naturally on Distributions

  [Hermanns, Krcal & Kretinsky @ CONCUR"14]

- Decentralized Bisimulation for Multiagent Systems

  [Song, Feng & Zhang @ AAMAS'14]

- Probabilistic Bisimulation for Realistic Schedulers

  [Eisentraut, Godskesen, Hermanns, Song & Zhang @ FM'15]

Thank you for your attention!